Spatio-Temporal Prediction of Wind Speed and Direction by Continuous Directional Regime

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***Abstract*—This paper proposes a statistical method for 1–6 hour-ahead prediction of hourly mean wind speed and direction to better forecast the power produced by wind turbines, an increasingly important component of power system operation. The wind speed and direction are modelled *via* the magnitude and phase of a complex vector containing measurements from multiple geographic locations. The predictor is derived from the spatio-temporal covariance which is estimated at regular time intervals from a subset of the available training data, the wind direction of which lies within a sliding range of angles centred on the most recent measurement of wind direction. This is a generalisation of *regime-switching* type approaches which train separate predictors for a few fixed regimes. The new predictor is tested on the Hydra dataset of wind across the Netherlands and compared to persistence and a cyclo-stationary Wiener filter, a state-of-the-art spatial predictor of wind speed and direction. Results show that the proposed technique is able to predict the wind vector more accurately than these benchmarks on dataset containing 4 to 27 sites, with greater accuracy for larger datasets.**

I. Introduction

Accurate short-term forecasts of wind power generation are essential for the reliable and economic operation of power systems, and will become increasingly important as the wind penetration on power systems around the world increases [1]–[4]. Wind power forecasts are used to inform the management of conventional plant and storage systems to maintain system stability while also reducing imbalance and under/over-production charges to wind generators making wind more competitive in the electricity marketplace.

The prediction technique employed depends greatly on the time scale of the required forecasts, for 6 hours to several days ahead Numerical Weather Predictors, very large atmospheric models, are required, but at shorter look-ahead times statistical methods are preferable [5]. A wide range of short-term forecasting techniques have been proposed and are summarised in [5], [6]. The power output of a wind farm often has a strong dependence on wind direction due to the arrangement of wind turbines or local terrain, however, few of these techniques utilise readily available measurements of direction or attempt to predict the future wind direction.

Direction has been captured in complex-valued neural networks, for example in [7]–[9], but these only model individual spatial locations. Others have developed regime-switching approaches which predict the wind speed depending on which direction-based regime the most recent measurements fall into, for example [10]–[12]. Two bivariate models are described in [13] predict speed and direction; the first is regime based and models the wind speed and direction, while the second models the perpendicular Cartesian components of the wind speed. All of these predictors are trained on a continuous series of the most recent measurements made at multiple locations. Other regime-switching approaches, such as the Markov-switching autoregressive model proposed in [14], show that superior forecasting performance can be achieved without basing regimes specific physical quantities.

Furthering other work on complex-valued prediction, reported in [15], this paper aims to extend the regime-switching type approaches, which commonly contain 2 or 3 fixed regimes (though the predictor for each regime is commonly adaptive) specific to the target prediction site. By introducing the concept of continuous directional regimes, we develop an adaptive spatial predictor which is optimised at regular intervals for the current wind conditions at multiple sites on a national scale based on the wind's behaviour during periods of similar conditions in the past.

The data model and approach to spatial prediction are introduced in Section II and the minimum mean squared error predictor and proposed continuous directional regime predictor are derived in Sections II-A and II-B. The testing procedure and results are presented in Section III and some conclusions drawn in IV.

II. Data Model and Spatial Prediction

At discrete time n, the wind speed and direction at M locations are embedded as the magnitude and phase of a complex valued vector $\boldsymbol{x}[n] \in \mathbb{C}^M$. The spatial covariance matrix is defined based on the expectation operator, $\mathcal{E}\{\cdot\}$, as $\mathbf{R}_{xx}[n,\tau] = \mathcal{E}\{\boldsymbol{x}[n]\boldsymbol{x}^{\mathrm{H}}[n-\tau]\}$. Where $\boldsymbol{x}^{\mathrm{H}}[n]$ denotes the Hermitian transpose of $\boldsymbol{x}[n]$ and τ is a general lag parameter.

It is well known that wind speed and wind direction are likely non-stationary (has time-varying probability distribution) and otherwise non-linear; both can be volatile and, direction particularly, can depend heavily on the physical characteristics of the measurement site. Furthermore, the seasonal and diurnal trends that characterize our human experience of the wind are themselves variable. In the succeeding text, we ignore the potential non-linear nature of the system and restrict ourselves to linear processing but drop the assumption of stationarity for a quasi-stationary behaviour, whereby the space-time covariance matrix can be assumed to be stationary—and therefore only dependent on the lag parameter τ—for sufficiently short time windows [16].

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$$
\begin{aligned}
\mathbf{R}_{ee}[n] &= \mathcal{E}\left\{(\boldsymbol{x}[n]-\boldsymbol{W}_n^{\mathrm{H}}\mathbf{x}_{n-\Delta})(\boldsymbol{x}^{\mathrm{H}}[n]-\mathbf{x}_{n-\Delta}^{\mathrm{H}}\boldsymbol{W}_n)\right\} \quad , \\
&= \mathbf{R}_{xx}[n,0]-\mathcal{E}\{\boldsymbol{x}[n]\mathbf{x}_{n-\Delta}^{\mathrm{H}}\}\boldsymbol{W}_n-\boldsymbol{W}_n^{\mathrm{H}}\mathcal{E}\{\mathbf{x}_{n-\Delta}\boldsymbol{x}^{\mathrm{H}}[n]\}+\boldsymbol{W}_n^{\mathrm{H}}\mathcal{E}\{\mathbf{x}_{n-\Delta}\mathbf{x}_{n-\Delta}^{\mathrm{H}}\}\boldsymbol{W}_n \quad , \\
&= \mathbf{R}_{xx}[n,0]-\mathbf{R}_{x\mathbf{x}}[n]\boldsymbol{W}_n-\boldsymbol{W}_n^{\mathrm{H}}\mathbf{R}_{x\mathbf{x}}^{\mathrm{H}}[n]+\mathbf{W}_n^{\mathrm{H}}\mathbf{R}_{\mathbf{xx}}[n]\boldsymbol{W}_n \quad ,
\end{aligned} \tag{5}
$$

where

$$
\mathbf{R}_{x\mathbf{x}}[n] = [\ \mathbf{R}_{xx}[n,\Delta]\ ,\quad \mathbf{R}_{xx}[n,\Delta{+}1]\ ,\quad \ldots\ ,\quad \mathbf{R}_{xx}[n,\Delta{+}N{-}1]\ ] \quad , \tag{6}
$$

$$
\mathbf{R}_{\mathbf{xx}}[n] = \begin{bmatrix} \mathbf{R}_{xx}[n{-}\Delta,0] & \mathbf{R}_{xx}[n{-}\Delta,1] & \ldots & \mathbf{R}_{xx}[n{-}\Delta,N{-}1] \\ \mathbf{R}_{xx}[n{-}\Delta{-}1,-1] & \mathbf{R}_{xx}[n{-}\Delta{-}1,0] & & \mathbf{R}_{xx}[n{-}\Delta{-}1,N{-}2] \\ \vdots & & \ddots & \vdots \\ \mathbf{R}_{xx}[n{-}\Delta{-}N{+}1,-N{+}1] & \mathbf{R}_{xx}[n{-}\Delta{-}N{+}1,-N{+}2] & \ldots & \mathbf{R}_{xx}[n{-}\Delta{-}N{+}1,0] \end{bmatrix}. \tag{7}
$$

A. MMSE Predictor

We consider the problem of predicting Δ samples ahead while minimising the mean-squared prediction error (MSE), based on M spatial measurements in $\boldsymbol{x}[n]$ and a time window containing N past samples for each site. Therefore, the prediction error can be formulated as

$$
\begin{aligned}
\boldsymbol{e}[n] &= \boldsymbol{x}[n]-\sum_{\nu=0}^{N-1}\boldsymbol{W}^{\mathrm{H}}[n,\nu]\boldsymbol{x}[n-\Delta-\nu] && (1) \\
&= \boldsymbol{x}[n]-\boldsymbol{W}_n^{\mathrm{H}}\mathbf{x}_{n-\Delta} \quad , && (2)
\end{aligned}
$$

with

$$
\boldsymbol{W}_n = \begin{bmatrix} \boldsymbol{W}[n,0] \\ \boldsymbol{W}[n,1] \\ \vdots \\ \boldsymbol{W}[n,N-1] \end{bmatrix} \in \mathbb{C}^{MN\times M} \quad , \tag{3}
$$

and

$$
\mathbf{x}_n = \begin{bmatrix} \boldsymbol{x}[n] \\ \boldsymbol{x}[n-1] \\ \vdots \\ \boldsymbol{x}[n-N+1] \end{bmatrix} \in \mathbb{C}^{MN} \quad . \tag{4}
$$

The matrices $\boldsymbol{W}[n,\nu] \in \mathbb{C}^{M\times M}$ describe the predictor's reliance on all spatial measurements taken $\nu{+}\Delta$ samples in the past, at time instance n. Specifically, $[\boldsymbol{W}[n,\nu]]_{p,q}$ addresses the influence of the measurement at site p onto the prediction at site q. In order to simply use the Hermitian transpose operator in (2), $\boldsymbol{W}[n,\nu]$ contains the complex conjugate prediction filter coefficients.

The error covariance matrix derived from (2), $\mathbf{R}_{ee}[n] = \mathcal{E}\{\boldsymbol{e}[n]\boldsymbol{e}^{\mathrm{H}}[n]\} \in \mathbb{C}^{M\times M}$, is obtained by taking expectations over the ensemble, and in itself may be varying with time n. Note that in case of stationarity, the dependency of both $\boldsymbol{W}_n$ and $\mathbf{R}_{ee}[n]$ on n vanishes. We will carry forward n since it is well known that the wind signal is non-stationary and develop an approximately stationary solution in Section II-B. Calculating $\mathbf{R}_{ee}[n]$ using (2) yields a quadratic expression in $\boldsymbol{W}_n$, Equation (5).

We assume that $\boldsymbol{x}[n]$ is stationary over at least 2Δ samples. As a result, $\mathbf{R}_{\mathbf{xx}}[n]$ is Hermitian and therefore positive semi-definite [17]. This property together with full rank of $\mathbf{R}_{\mathbf{xx}}[n]$ admits a unique solution to minimises the mean square error,

$$
\boldsymbol{W}_{n,\mathrm{opt}} = \arg\min_{\boldsymbol{W}_n} \mathrm{trace}\{\mathbf{R}_{ee}[n]\} \quad . \tag{8}
$$

It can be shown that $\mathrm{trace}\{\mathbf{R}_{ee}[n]\}$ is quadratic in $\boldsymbol{W}_n$, such that the solution to (8) can be found by matrix- and complex-valued calculus [18]. Finding the minimum requires equating the gradient w.r.t. the unconjugated predictor coefficients in $\boldsymbol{W}_n^*$ to zero. We utilise results from [18] which show that for constant matrices $\mathbf{A}$ and $\mathbf{B}$ the expressions $\partial\mathrm{trace}\{\mathbf{A}\boldsymbol{W}_n^{\mathrm{H}}\mathbf{B}\}/(\partial\boldsymbol{W}_n^*) = \mathbf{BA}$ but $\partial\mathrm{trace}\{\mathbf{A}\boldsymbol{W}_n\mathbf{B}\}/(\partial\boldsymbol{W}_n^*) = \mathbf{0}$ hold. Applying this, and using the product rule for differentiation of the quadratic term in (5), yields

$$
\frac{\partial}{\partial\boldsymbol{W}_n^*}\mathrm{trace}\{\mathbf{R}_{ee}[n]\} = -\mathbf{R}_{x\mathbf{x}}^{\mathrm{H}}[n]+\mathbf{R}_{\mathbf{xx}}[n]\boldsymbol{W}_n \quad . \tag{9}
$$

Finally, setting the gradient on the r.h.s. of (9) equal to zero yields the optimum predictor coefficients that minimise $\mathrm{trace}\{\mathbf{R}_{ee}[n]\}$,

$$
\boldsymbol{W}_{n,\mathrm{opt}} = \mathbf{R}_{\mathbf{xx}}^{-1}[n]\mathbf{R}_{x\mathbf{x}}^{\mathrm{H}}[n] \quad , \tag{10}
$$

which is the well-known Wiener-Hopf solution [19], [20].

B. Continuous Directional Regimes

The time dependent covariance matrix $\mathbf{R}_{xx}[n,\tau]$ is estimated by including only historic data for periods when the wind direction was similar to that of, or in the same *directional regime* as, the most recent measurements. A *continuous directional regime* refers to the sliding range of angles, 2Θ, centred on the most recent measurement of wind direction. The multiple mini-series of $N+1$ samples (corresponding to the concatenation of $\boldsymbol{x}[n]$ and $\mathbf{x}_{n-\Delta}$ in (2)) contributing to the estimation of $\boldsymbol{W}_{n,\mathrm{opt}}$ are assumed to be jointly stationary.

Each historic measurement $\boldsymbol{x}[i]$ that is in the same directional regime of the most recent measurement must be accompanied by its N preceding samples which may not lie within the current regime, therefore we define $\mathbf{R}_{xx}[n,\delta,\tau] = \mathbf{R}_{xx}[n-\delta,\tau]$ before proceeding.

The estimation of the spatial covariance matrix can now be written as

$$
\mathbf{R}_{xx}[n,\delta,\tau] = \frac{1}{|P[n]|}\sum_{i\in P[n]}\boldsymbol{x}[i-\delta]\boldsymbol{x}^{\mathrm{H}}[i-\delta-\tau] \quad , \tag{11}
$$

where $P[n]$ is the set of time indexes, p, that satisfy $|\arg \boldsymbol{x}[p] - \overline{\arg \boldsymbol{x}[n]}) \mod (-\pi, \pi])| < \Theta$, where $\arg \boldsymbol{x}[i] \in (-\pi, \pi]$ and $\overline{\arg \boldsymbol{x}[n]}$ denotes the circular mean of $\arg \boldsymbol{x}[n]$.

By estimating the spatial covariance for a specific directional regime, the propagation of changes in wind speed and directional from upwind to downwind sites can be captured. The inclusion of mismatched information corresponding to periods during which the wind direction was significantly different to the present, which would have the effect of smoothing, or at least skewing the directional dependence of the predictor, is avoided.

The regime specific optimal predictor can be recalculated at each time step or at regular intervals to reduce computational time at little cost in accuracy.

III. Testing and Results

The proposed method will be compared to the cyclo-stationary Wiener filter (CsWF) [15] and persistence, a common benchmark for wind prediction. Persistence predicts that the future wind speed will be the same as the most recent measurement. Like the direction based approach described in this paper, the CsWF also make a quasi-stationary assumption but this time based on the cyclic seasonal behaviour of the wind; the space-time covariance is estimated using historic data form the same season as the current prediction.

The cyclo-stationry covariance matrix is estimated as

$$\hat{\mathbf{R}}_{xx}[n,\tau] = \frac{1}{K(L+1)} \sum_{k=1}^{K} \left(\sum_{\nu=\frac{L}{2}}^{\frac{L}{2}} \boldsymbol{x}[n-kT-\nu]\boldsymbol{x}^{\mathrm{H}}[n-kT-\nu-\tau] \right) + \frac{2}{L} \sum_{\nu=1}^{\frac{L}{2}} \boldsymbol{x}[n-\nu]\boldsymbol{x}^{\mathrm{H}}[n-\nu-\tau] \quad , \quad (12)$$

where L is the length of each cyclo-stationary window, K is the number years of training data being used, and T is the period of the cyclo-stationariry, i.e. 1 year. For the dataset in question, $L = 20$ weeks was found to be optimal and $K = 5$ to make use of all available training data.

A. Test Data

The data used for testing is from the Hydra dataset of hourly mean potential wind at multiple locations across the Netherlands, shown in Fig. 1. Data from 2001–2005 inclusive is used as training data and data from 2006 is used for testing.

The measured wind speed has been corrected for the effects of shelter from buildings or vegetation. The resulting *potential* wind is an estimate of the wind speed that could have been measured at 10m height if the station's surroundings were free of obstacles and flat with a roughness length equal to that of grass onshore (0.03m) and water offshore (0.002m). For more information on this process see [21].

This transformation aids spatial prediction by removing biases present at individual measurement locations that would otherwise interfere with the spatio-temporal correlation of the data. The procedure is simple to implement once information regarding the terrain surrounding a weather station is known.

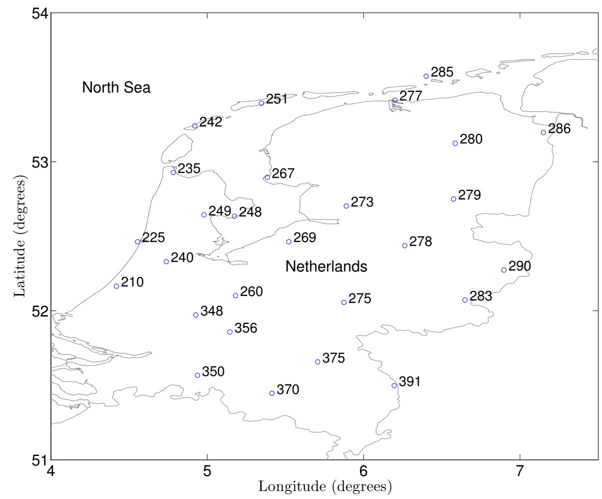

Fig. 1. Map of the Netherlands showing the location of weather stations and their reference numbers.

In order to assess the performance of the proposed predictor on spatial datasets of different sizes, it is tested first on 4 central locations and then on larger datasets with sites are added progressively beginning with those closest to the original 4.

B. Results

The range of wind direction in a regime, Θ, is take to be $\frac{2\pi}{3}$ since the performance at this range is found by numerical testing to yield better results than $\frac{\pi}{3}$ and π. Given the large range of wind direction, the improvement in prediction is perhaps best thought of as due to the exclusion of mismatched data, rather than the inclusion of well matched data. The number of historic samples, $|P|$, that contribute to the estimation of the covariance matrix for a given regime ranges from 33230 to 37043 depending on sits in the data model and the wind direction.

The number of past time samples N is taken to be 3 since any more significantly increases the computational complexity for negligible reduction in prediction error. For the same reason, the covariance matrix is only recalculated every 24 time steps, i.e. once per day.

The performance of the 1-hour-ahead ($\Delta = 1$) forecast in terms of root mean squared error (RMSE) for the CDR and CsWF predictors is plotted in Fig. 2 for data models containing information from between 4 and 27 sites. The proposed CDR predictions are consistently more accurate than the CsWF, but only by a small margin.

There is a clear reduction in RMSE at all prediction locations as the amount of spatial information is increased. Particularly large improvements are seen at specific sites when new data from nearby locations is added; for example, at site 260 when sites 240, 248 and 256 are added to the data model. Site 249 also sees marked improvement when a number of surrounding sites are included in the data model.

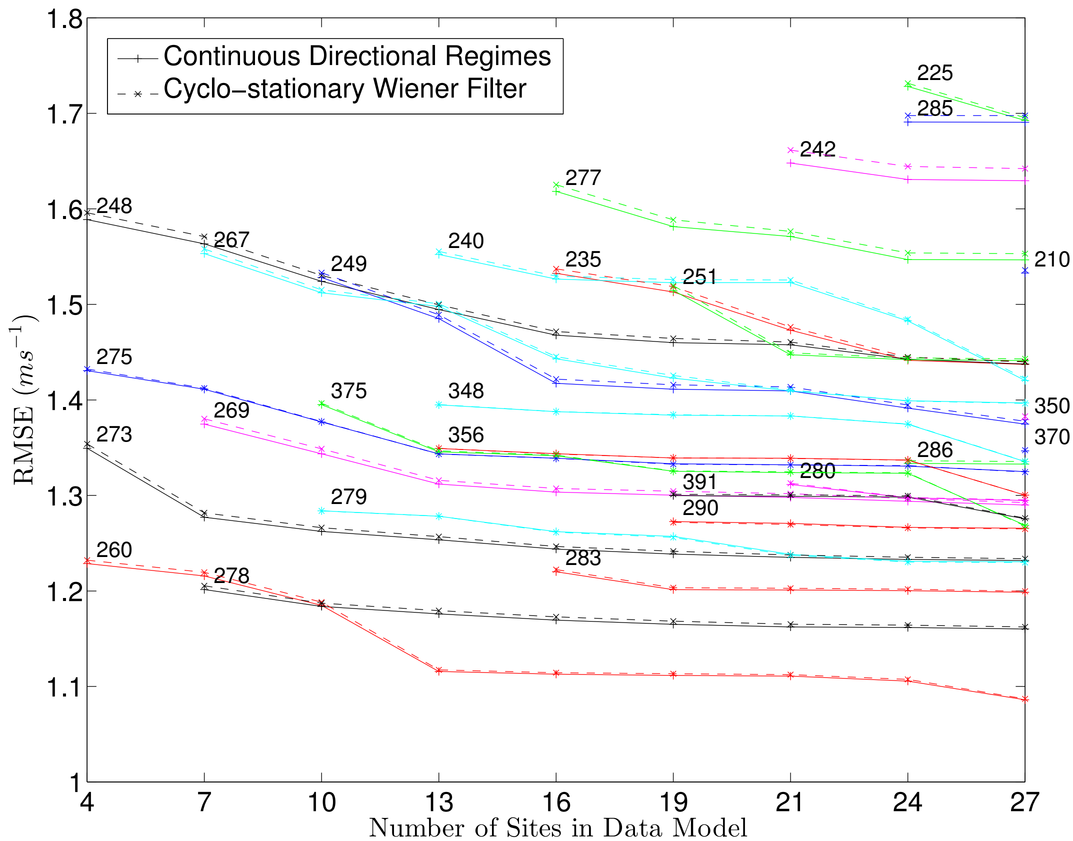

Fig. 2. Root Mean Squared Error (RMSE) for 1-hour-ahead forecast at sites, labelled by station number, for data models containing 4 to 27 sites.

The performance of the directional predictor is compared to persistence for look-ahead times from 1 to 6 hours in Table I. The CDR is an improvement on persistence at all look-ahead times, with approximately twice the reduction in RMSE for the 27 site data model compared to that containing only 4 sites.

IV. Conclusions

This paper proposes a new spatio-temporal predictor for hourly mean wind speed and direction at multiple measurement locations. Inspired by approaches which define fixed, discrete *regimes* based on wind direction, an adaptive predictor based on continuous direction regimes (CDR) is derived and tested, and shown to produce accurate forecasts for look-ahead times of 1 to 6 hours.

The CDR is a spatial covariance-based minimum MSE predictor, it is innovative in its selection of training data in real time to exclude mismatched historic data based on the most recent measurements. The spatial covariance matrix is estimated using only data from periods during which the wind direction was within a fixed range of its present direction from which the adaptive predictor is calculated.

The new predictor is tested on the Hydra dataset and compared to persistence and the cyclo-stationary Wiener filter, another spatial-covariance-based adaptive predictor. The CDR

			Data Model	
Location	Δ	Persistence	4 Sites	27 Sites
248: Wijdenes	1	1.68	1.59	1.44
	2	2.26	2.10	1.87
	3	2.69	2.50	2.22
	4	3.05	2.83	2.54
	5	3.36	3.12	2.84
	6	3.63	3.36	3.10
260: De Bilt	1	1.37	1.23	1.09
	2	1.73	1.53	1.34
	3	2.03	1.79	1.58
	4	2.28	2.01	1.80
	5	2.51	2.21	2.00
	6	2.70	2.37	2.18
273: Marknesse	1	1.55	1.35	1.23
	2	2.05	1.74	1.53
	3	2.46	2.09	1.82
	4	2.81	2.39	2.10
	5	3.11	2.66	2.37
	6	3.37	2.88	2.61
275: Deelen	1	1.66	1.43	1.32
	2	2.13	1.78	1.61
	3	2.51	2.09	1.88
	4	2.82	2.35	2.14
	5	3.10	2.59	2.37
	6	3.34	2.80	2.58

TABLE I. Comparison of CDR Root Mean Squared Error at the 4 sites in the smallest data model to persistence and when included in a larger data model at look-ahead times from $\Delta = 1$ to 6 hours.

is found to produce forecasts which are a significant improvement on persistence and consistently more accurate than the CsWF, if only by a small margin. Furthermore, it is shown that the prediction error is reduced as more spatial information is added to the data model.

While it is relatively crude, the proposed method performs well and provide encouraging support for the future refinement of this type of approach, perhaps building in constraints on wind speed or choosing specific measurement sites to improve prediction at some target location.

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